TECHNIQUE FOR ESTIMATING OPTIMUM SIZE AND SHAPE OF PLOT FROM FERTILIZER TRIAL DATA : A MODIFIED APPROACH

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(Received : September, 1975)

In a recent article entitled "Technique for Estimating Optimum Size and Shape of Plot" (Ray, S., *et. al.* 1973), the authors have suggested a useful method of utilizing the data from manurial trials to reconstruct it as one which can be treated as data from uniformity trial. Briefly, the technique of converting the data follows from the simple model:

$$y_{ijk}=\mu+\tau_i+\beta_j+\epsilon_{ijk} \quad \ldots(1)$$

where

y_{ijk} is the yield of the (ijk)th plot, μ the general mean, τ_i the ith treatment effect and ϵ_{ijk} being the error random variable having a normal distribution with zero mean and constant variance. Since the block effects do not come into picture in case of uniformity trials, the same is ignored in model (1), leading to a modified model:

$$y_{ijk}=\mu+\tau_i+\epsilon^*_{ijk} \quad \ldots(2)$$

The suggested estimator for the treatment effect τ_i is t_i which is defined as

$$t_i=\frac{T_i}{\gamma}-m \quad \ldots(3)$$

in which T_i is the total of r observations effected by the treatment effect τ_i and m the general mean which estimates the parameter μ. The estimated value t_i is next subtracted from each plant yield Y effected by the ith treatment; the resulting residuals of the type $(y-t)$ will then produce a kind of data which can be analyzed in the same way as done in uniformity trials.

The point of interest and of concern is the suggested estimator t_i for τ_i. In case of Complete Block Designs like *RCBD*, t_i works well and in fact does not contain any block effects, since all treatments appear in each block. But in case of data from an incomplete block design or a confounded design—like the one used by the authors in their illustrative case—the estimate t_i will not be free of block effects and as a consequence of this, the residual data of the type $(y-t)$ besides being adjusted for the treatment effect τ_i, would also be rid of some block effects.

In uniformity trials, as we know, the variation in soil fertility is a chief source of variation and determines to a large extent optimum size and shape of plots. In particular, in field trials, the block effects chiefly contain information about the soil fertility variations and therefore any removal of these block effects from the residual data would naturally affect the pertinent information about soil fertility variations and hence would affect in turn the efficiency in determining the optimum shape and size of plot.

The main theme then of this paper is to propose an alternative estimator for τ_i in place of the above t_i so that it should not only not contain any block effect and at the same time leave the residual data of the type $(y-t)$ to contain most information about variations due to extraneous sources—random as well as block effects. The emphasis, however, will be more on the algebraic form rather than on any numerical illustration.

Hence by way of illustration of the proposed method, let us consider a 2^3 Confounded design (balanced) with partial information about all interaction effects. Table I sets forth such a design which confounds AB in Rep I, AC in Rep II, BC in Rep III and ABC in Rep IV.

TABLE I

A 2^3 Confounded Design with Equal Partial Information on all Interactions

Rep I		*Rep II*		*Rep III*		*Rep IV*	
Block 1	*Block 2*	*Block 3*	*Block 4*	*Block 5*	*Block 6*	*Block 7*	*Block 8*
y_{000} (1)	y_{010} (2)	y_{000} (3)	y_{001} (4)	y_{000} (5)	y_{001} (6)	y_{000} (7)	y_{001} (8)
y_{001} (1)	y_{011} (2)	y_{010} (3)	y_{011} (4)	y_{011} (5)	y_{010} (6)	y_{011} (7)	y_{010} (8)
y_{110} (1)	y_{100} (2)	y_{101} (3)	y_{100} (4)	y_{100} (5)	y_{101} (6)	y_{101} (7)	y_{100} (8)
y_{111} (1)	y_{101} (2)	y_{111} (3)	y_{110} (4)	y_{111} (5)	y_{110} (6)	y_{110} (7)	y_{111} (8)
AB		*AC*		*BC*		*ABC*	

A cell value $y_{ijk(l)}$ in each replication represents a response observation on per plot basis pertaining to the treatment combination $a_i\, b_j\, c_k$ $(i, j, k=0, 1)$ in block l $(l=1, 2, ..., 8)$ and is a total of n observations, one for each of n plants in every plot.*

Under the assumptions of a customary linear model to hold good, an observation of the type $y_{ijk(l)}$ may be represented by

$$y_{ijk(l)}=[\mu+\alpha_i+\beta_j+\gamma_k+(\alpha\beta)_{ij}+(\alpha\gamma)_{ik}+(\beta\gamma)_{jk}+(\alpha\beta\gamma)_{ijk}] \qquad ...(4)$$
$$+\delta_l+\epsilon_{ijk(l)}$$

with

$$\sum_i \alpha_i=\sum_j \beta_j=\sum_k \gamma_k=0,$$

$$\sum_i (\alpha\beta)_{ij}=0 \text{ etc.}$$

Here α_i etc., are the main effects, $(\alpha\beta)_{ij}$ etc., are the interaction effects, δ_l is the effects of the block l in which treatment $a_i\, b_j\, c_k$ appears; $\epsilon_{ijk(l)}$ represents the random error component in $y_{ijk(l)}$ observation.

For convenience, the above can be rewritten as

$$y_{ijk(l)}=\mu_{ijk}+\delta_l+\epsilon_{ijk(l)} \qquad ...(5)$$

with μ_{ijk} replacing the evpression in the square bracket in (4).

If we designate the treatment total for the treatment combination $a_i\, b_j\, c_k$ by $y_{ijk(\cdot)}$, we notice that such totals will not be free of block effects, since all treatment combinations do not appear in all blocks. In fact, we have

$$y_{ijk(\cdot)}=4\mu_{ijk}+B_{(\cdot)}+\epsilon_{ijk(\cdot)} \qquad ...(6)$$

where $B_{(\cdot)}$ represents the total of only those block effects in which the treatment $a_i\, b_j\, c_k$ appears; $\epsilon_{ijk(\cdot)}$ being a similar total for error term (incidentally it may be noted that this is equivalent to the total T_i suggested by Ray, *et. al.*, in their article.). Hence the mean $\bar{y}_{ijk(\cdot)}$ obtained by dividing this total by 4 (in the present illustrative case) will also contain a block effect equal to $1/4B_{(\cdot)}$. For instance, the treatment combination $a_1\, b_0\, c_1$ appears in blocks 2, 3, 6, and 7 in Table I. Hence the observed mean $\bar{y}_{101(\cdot)}$ also contains a block effect equal to $1/4\ (\delta_2+\delta_3+\delta_6+\delta_7)$, besides the treatment effect μ_{ijk} plus the random component $\overline{\epsilon_{ijk(\cdot)}}$.

Thus if we can reconstruct an estimator, sas $\bar{y}'_{ijk(\cdot)}$, which does not contain this block effect, $\bar{y}'_{ijk(\cdot)}$ will estimate μ_{ijk}, If then $y_{ijk(l)}m$ $(m=1, 2, ..., n)$ represents an observation on mth individual

*The method presumes that yield figures are recorded plant-wise. However, for estimating treatment effects, the total $y_{ijk(l)}$ is utilized.

plant in $(i\,j\,k)$th plot receiving the treatment $a_i\,b_j\,c_k$ in block l ($l=1, 2, \ldots, 8$), the residual data $x_{ijk(l)m}$ defined by

$$x_{ijk(l)m}=y_{ijk(l)m}-\bar{y}'_{ijk(\cdot)} \qquad \ldots(7)$$

will contain information on all extraneous factors—random as well as block effects, mostly due to soil variation. And $x_{ijk(l)m}$ computed on plant-wise basis for all observations in the experiment will produce a kind of data-base which can be treated as done in uniformity trial data.

It may be readily verified that for the above 2^3 design (Table I), the following quantity $\bar{y}'_{ijk(\cdot)}$ will estimate μ_{ijk}, freed of block effects (*see* Federer, 1967).

$$\bar{y}'_{ijk(\cdot)}=\tfrac{1}{4}\left[\frac{A_i+B_j+C_k}{4}+\frac{(AB)'_{i+j}+(AC)'_{i+k}+(BC)'_{j+k}+(ABC)'_{i+j+k}}{3}-\frac{3G}{4}\right] \qquad \ldots(8)$$

where, in the modular notation,*

A_i=the total of all observations at level a_i of factor A, with similar definitions for B_j, C_k

$(AB)'_{i+j}$=sum of all observations on treatment combinations which satisfy the relationship $x_1+x_2=i+j$ (Mod 2) and which appear in blocks providing partial information on AB. And similar definitions for $(AC)'_{i+k}$ & $(BC)'_{j+k}$

$(ABC)'_{i+j+k}$=sum of all observations on treatment combinations which satisfy the relationship $x_1+x_2+x_3=i+j+k$ (Mod 2) and which appear in blocks providing partial information on ABC.

G=Grand total based on all the observations.

Obviously formula (8) undergoes change for any change in the design.

For all routinely employed confounded and other incomplete block designs in field experiments in agriculture, the formulae for adjustments to treatment means for block effects, similar to the one in (8), can be constructed so that the block effects are eliminated from the treatment means before generating the residual components of the type $X_{ijk(l)m}$ using the expression (7). Once the residual data is thus generated, it serves as a data-base, similar to the one of uniformity trial data, and hence can be treated in the usual way for determining the optimum shape and size of plot.

*In the usual modular notation x_1, x_2, x_3, each takes on valnes (0,1).

ACKNOWLEDGEMENT

The author is grateful to the referee for a few valuable comments which improved the quality of this paper. He is also grateful to Dr. R. Narayana, Director of Instruction (BS&H), for his constant encouragement during this work.

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